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The figure below is not a convex set

as the line segment joining PQ does not lie on the set.

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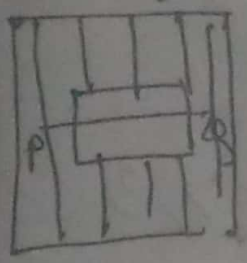
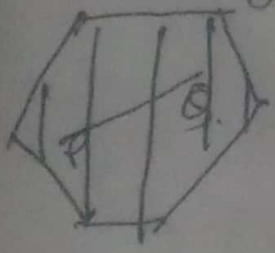


As line segment joining PQ does not lie on the set,

The figure below is a convex set.



If we take any line segment joining P, Q where P, Q are two points on the set lie entirely on the set..

Show that

Ex $X = \{(x_1, x_2) \mid 9x_1^2 + 4x_2^2 \leq 36\}$ is a convex set.

Let $(x_1, x_2) \in X$. Then $9x_1^2 + 4x_2^2 \leq 36$

Let $(y_1, y_2) \in X$ then $9y_1^2 + 4y_2^2 \leq 36$

Now $u = \lambda(x_1, x_2) + (1-\lambda)(y_1, y_2)$

$= (\lambda x_1 + (1-\lambda) y_1, \lambda x_2 + (1-\lambda) y_2)$

is the convex combination of the points (x_1, x_2) and (y_1, y_2) where $0 \leq \lambda \leq 1$

Now $9[\lambda x_1 + (1-\lambda) y_1]^2 + 4[\lambda x_2 + (1-\lambda) y_2]^2$

$= 9[\lambda^2 x_1^2 + 2\lambda(1-\lambda) x_1 y_1 + (1-\lambda)^2 y_1^2] +$

$4[\lambda^2 x_2^2 + 2\lambda(1-\lambda) x_2 y_2 + (1-\lambda)^2 y_2^2]$

$= \lambda^2 (9x_1^2 + 4x_2^2) + (1-\lambda)^2 [9y_1^2 + 4y_2^2]$

$+ 2\lambda(1-\lambda)(9x_1 y_1 + 4x_2 y_2)$

$\leq \lambda^2 \times 36 + (1-\lambda)^2 \times 36 + 2\lambda(1-\lambda) \times 36$

$= 36[\lambda + (1-\lambda)]^2 = 36.$

[$\because (x_1 - y_1)^2 \geq 0 \quad \forall x_1, y_1$

$\Rightarrow x_1^2 + y_1^2 - 2x_1 y_1 \geq 0 \Rightarrow 2x_1 y_1 \leq x_1^2 + y_1^2$

i.e. $x_1 y_1 \leq \frac{x_1^2 + y_1^2}{2}$

Similarly $2x_2 y_2 \leq x_2^2 + y_2^2$

$\Rightarrow x_2 y_2 \leq \frac{x_2^2 + y_2^2}{2}$

So $9x_1 y_1 + 4x_2 y_2 \leq 9\frac{(x_1^2 + y_1^2)}{2} + 4\frac{(x_2^2 + y_2^2)}{2}$

$= \frac{(9x_1^2 + 4x_2^2) + (9y_1^2 + 4y_2^2)}{2} \leq \frac{36 + 36}{2} = 36$]

Hence $u \in X$. So X is a convex set.

~~(ii) $X = \{(x_1, x_2) \mid x_1 \leq 5, y_1$~~

(ii) $X = \{(x_1, x_2) \mid x_1 \leq 5, x_2 \geq 3\}$ is a convex set.

Let $u = (x_1, x_2) \in X$ and $v = (y_1, y_2) \in X$

$x_1 \leq 5, x_2 \geq 3$ and $y_1 \leq 5, y_2 \geq 3$

Now $\lambda u + (1-\lambda)v = (\lambda x_1 + (1-\lambda)y_1, \lambda x_2 + (1-\lambda)y_2)$ is the convex combination of u, v if $0 \leq \lambda \leq 1$

Now $\lambda x_1 + (1-\lambda) y_1 \leq 5\lambda + 5(1-\lambda) = 5$

$\lambda x_2 + (1-\lambda) y_2$

$\geq 3\lambda + 3(1-\lambda) = 3$

Hence $\lambda u + (1-\lambda)v \in X$

Hence X is a convex set.

H.W.

1) S.T. $X = \{(x_1, x_2) \mid x_1 + 2x_2 \leq 5\}$ is a convex set.

2) Show that $X = \{(x_1, x_2) \mid x_1 + x_2 \leq 50, x_1 + 2x_2 \leq 80, 2x_1 + x_2 \geq 20, x_1, x_2 \geq 0\}$ is a convex set.